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On the Nonlinear Mixed Sensitivity Problem *

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Abstract

In this note we discuss some examples of the nonlinear H^∞ design theory developed by Foias and Tannenbaum [9], [11] as well as an extension to the mixed sensitivity problem.

1 Introduction

This note will be concerned with an extension of the nonlinear H^∞ synthesis theory developed in [9], [11] for the sensitivity minimization problem to the 2 block mixed sensivity method. We should note that we have developed a computer program at Honeywell by which we can actually implement this procedure.

Our synthesis method is valid for majorizable input/output operators (and can be extended to those operators which are approximable by such). See Section 2 for the precise definition. In particular, we consider operators which are analytic in a ball around the origin in a complex Hilbert space. It turns out that it is possible to express each n-linear term of the Taylor expansion of such an operator as a linear operator on a certain tensor space. (Our class of operators also include Volterra series of fading memory [7].) This allows us to iteratively apply the classical commutant lifting theorem in designing a compensator. We call our method the *iterative commutant lifting procedure*. See Section 6 and [9], [11].) For single input/single output (SISO) systems, this leads to the construction of a compensator which is optimal relative to a certain sensitivity function which will be defined below. Moreover in complete generality (i.e. for multiple input/multiple ouput (MIMO) systems), our procedure will ameliorate (in the sense of our nonlinear weighted sensitivity criterion), any given design. We note that for linear systems, our method reduces to the standard H^∞ design technique as discussed for example in [13] and [16].

In developing the present theory, we have had to extend some of the skew Toeplitz techniques of [6], [10], and [12] to linear operators defined on certain tensor spaces. This has lead to several novel results in computational operator theory, and for example provides a way of iteratively constructing the nonlinear intertwining dilation of the nonlinear commutant lifting theorem considered in [2] and [3].

2 Analytic Mappings on Hilbert Space

In order to carry out our extension of H^∞ synthesis theory to nonlinear systems, we will need to first discuss a few standard results about analytic mappings on Hilbert spaces. See [9] and [11] for complete details.

Let G and H denote complex Hilbert spaces. Set

$$B_{r_o}(G) := \{g \in G : \|g\| < r_o\}$$

(the open ball of radius r_o in G about the origin). Then we say that a mapping $\phi : B_{r_o}(G) \to H$ is *analytic* if the complex function $(z_1, \dots, z_n) \mapsto \langle (z_1 g_1 + \dots + z_n g_n), h\rangle$ is analytic in a neighborhood of $(1, 1, \dots, 1) \in \mathbf{C}^n$ as a function of the complex variables $z_1, \dots, z_n$ for all $g_1, \dots, g_n \in G$ such that $\|g_1 + \dots + g_n\| < r_o$, for all $h \in H$, and for all $n > 0$. (Note that we denote the Hilbert space norms in G and H by $\|\ \|$ and the inner products by $\langle , \rangle$.)

We will now assume that $\phi(0) = 0$. It is easy to see that if $\phi : B_{r_o}(G) \to H$ is analytic, then ϕ admits a convergent Taylor series expansion, i.e.

$$\phi(g) = \phi_1(g) + \phi_2(g, g) + \cdots + \phi_n(g, \cdots, g) + \cdots$$

where $\phi_n : G \times \cdots \times G \to H$ is an n-linear map. Clearly, without loss of generality we may assume that the n-linear map $(g_1, \cdots, g_n) \to \phi(g_1, \cdots, g_n)$ is symmetric in the arguments $g_1, \cdots, g_n$. This assumption will be made throughout this paper for the various analytic maps which we consider. For ϕ a Volterra series, ϕ_n is basically the n^{th}-Volterra kernel.

Now set

$$\hat{\phi}_n(g_1 \otimes \cdots \otimes g_n) := \phi_n(g_1, \cdots, g_n).$$

Then $\hat{\phi}_n$ extends in a unique manner to a dense set of $G^{\otimes n} := G \otimes \dots \otimes G$ (tensor product taken n times). Notice by $G^{\otimes n}$ we mean the Hilbert space completion of the algebraic tensor product of the G's. Clearly if $\hat{\phi}_n$ has finite norm on this dense set, then $\hat{\phi}_n$ extends by continuity to a bounded linear operator $\hat{\phi}_n : G^{\otimes n} \to H$. By abuse of notation, we will set $\phi_n := \hat{\phi}_n$.

We now conclude this section with two key definitions.

Definitions 1.

(i) Notation as above. By a *majorizing sequence* for the holomorphic map ϕ, we mean a positive sequence of numbers α_n $n = 1, 2, \dots$ such that $\|\phi_n\| < \alpha_n$ for $n \geq 1$. Suppose that $\rho := \limsup {\alpha_n}^1/n < \infty$. Then it is completely standard that the Taylor series expansion of ϕ converges at least on the ball $B_r(G)$ of radius $r = 1/\rho$.

(ii) If ϕ admits a majorizing sequence as in (i), then we will say that ϕ is *majorizable.*

3 Control Theoretic Preliminaries

We start here with the control problem definition. In Section 7, we will extend this set-up to the nonlinear mixed sensitivity case. First, we will need to consider the precise kind of input/output operator we will be considering. Once again we are following [9] and [11] here.

We will assume that all of the operators we consider are causal and majorizable. Throughout this note $H^2(\mathbf{C^k})$ will denote the standard Hardy space of $\mathbf{C^k}$-valued functions on the unit circle (k may be infinite, i.e., in this case $\mathbf{C^k}$ is replaced by h^2, the space of one-sided square summable sequences). We now make the following definition:

Definition 2.

Let $S : H^2(\mathbf{C^k}) \to H^2(\mathbf{C^k})$ denote the canonical unilateral right shift. Then we say an input/output operator ϕ is *locally stable* if it is causal and majorizable, $\phi(0) = 0$, and if there exists an $r > 0$ such that $\phi : B_r(H^2(\mathbf{C^k})) \to H^2(\mathbf{C^k})$ with $S\phi = \phi \circ S$ on $B_r(H^2(\mathbf{C^k}))$. We set

$$C_l := \{\text{space of locally stable operators}\}.$$

Since the theory we are considering is local, the notion of local stability is sufficient for all of the applications we have in mind. The interested reader can compare this notion, with the more global notions of stability as for example discussed in [5].

The theory we are about to give holds for all plants which admit coprime locally stable factorizations. However, for simplicity we will assume that our plant is also locally stable. Accordingly, let P, W denote locally stable operators, with W invertible. In a typical feedback system [16], P represents the plant, and W the weight or filter on the set of disturbances whose energy is bounded by 1. Now we say that the feedback compensator C *locally stabilizes* the closed loop if the operators $(I + P \circ C)^{-1}$ and $C \circ (I + P \circ C)^{-1}$ are well-defined and locally stable. By a result of [1], C locally stabilizes the closed loop if and only if

$$C = \hat{q} \circ (I - P \circ \hat{q})^{-1} \tag{1}$$

for some $\hat{q} \in C_l$. Notice then that the weighted sensitivity (see [13] and [16] for all the relevant engineering definitions and motivation), $(I + P \circ C)^{-1} \circ W$ can be written as $W - P \circ q$, where $q := \hat{q} \circ W$. (Since W is invertible, the data q and $\hat{q}$ are equivalent.) In this context, we will call such a q, a *compensating parameter*. Note that from the compensating parameter q, we get a locally stabilizing compensator C via the formula (1).

The problem we would like to solve here, is a version of the classical disturbance attenuation problem of [13], [16]. This of course corresponds to the "minimization" of the "sensitivity" $W - P \circ q$ taken over all locally stable q. In order to formulate a precise mathematical problem, we need to say in what sense we want to minimize $W - P \circ q$. This we will do in the next section where we will propose a notion of "sensitivity minimization" which we seems quite natural to analytic input/output operators.

4 Sensitivity Function

In this section we define a fundamental object, namely a nonlinear version of *sensitivity*. We will see that while the optimal H^∞ sensitivity is a real number in the linear case, the measure of performance which seems to be more natural in this nonlinear setting is a certain function defined in a real interval. This new kind of performance criterion is one of the keys concepts developed in [9] and [11].

In order to define our notion of sensitivity, we will first have to partially order germs of analytic mappings. All of the input/output operators here will be locally stable. We also follow here our convention that for given $\phi \in C_l$, ϕ_n will denote the bounded linear map on the tensor space $(H^2(\mathbf{C^k}))^{\otimes n}$ associated to the n-linear part of ϕ which we also denote by ϕ_n (and which we always assume without loss of generality is symmetric in its arguments). The context will always make the meaning of ϕ_n clear.

We can now state the following definitions:

Definitions 3.

(i) For $W, P, q \in C_l$ (W is the weight, P the plant, and q the compensating parameter), we define the *sensitivity function* $S(q)$,

$$S(q)(\rho) := \sum_{n=1} \rho^n \|(W - P \circ q)_n\|$$

for all $\rho > 0$ such that the sum converges. Notice that for fixed P and W, for each $q \in C_l$, we get an associated sensitivity function.

(ii) We write $S(q) \preceq S(\tilde{q})$, if there exists a $\rho_o > 0$ such that $S(q)(\rho) \leq S(\tilde{q})(\rho)$ for all $\rho \in [0, \rho_o]$. If $S(q) \preceq S(\tilde{q})$ and $S(\tilde{q}) \preceq S(q)$, we write $S(q) \cong S(\tilde{q})$. This means that $S(q)(\rho) = S(\tilde{q})(\rho)$ for all $\rho > 0$ sufficiently small, i.e. $S(q)$ and $S(\tilde{q})$ are equal as germs of functions.

(iii) If $S(q) \preceq S(\tilde{q})$, but $S(\tilde{q}) \not\preceq S(q)$, we will say that q *ameliorates* $\tilde{q}$. Note that this means $S(q)(\rho) < S(\tilde{q})(\rho)$ for all $\rho > 0$ sufficiently small.

Now with Definitions 3, we can define a notion of "optimality" relative to the sensitivity function:

Definitions 4.

(i) $q_o \in C_l$ is called *optimal* if $S(q_o) \preceq S(q)$ for all $q \in C_l$.

(ii) We say $q \in C_l$ is *optimal with respect to its n-th term* q_n, if for every n-linear $\hat{q}_n \in C_l$, we have

$$S(q_1 + \ldots + q_{n-1} + q_n + q_{n+1} \ldots) \preceq S(q_1 + \ldots + q_{n-1} + \hat{q}_n + q_{n+1} + \ldots).$$

If $q \in C_l$ is optimal with respect to all of its terms, then we say that it is *partially optimal*.

5 Iterative Commutant Lifting Method

In this section, we discuss the main construction of this paper from which we will derive both partially optimal and optimal compensators relative to the sensitivity function given in Definitions 3 above. As before, P will denote the plant, and W the weighting operator, both of which we assume are locally stable. As in the linear case, we always suppose that P_1 is an isometry, i.e. P_1 is inner. In order to state our results, we will need a few preliminary remarks and to set-up some notation. We refer the interested reader to [11], [9] for the precise proofs of the various results in this section.

We begin by noting the following key relationship:

$$(W - P \circ q)_k = W_k - \sum_{1 \leq j \leq k} \sum_{i_1 + \ldots + i_j = k} P_j(q_{i_1} \otimes \ldots \otimes q_{i_j})$$

Note that once again for ϕ of fading memory, ϕ_n denotes the n-linear part of ϕ, as well as the associated linear operator on the appropriate tensor space.

We are now ready to formulate the *iterative commutant lifting procedure*. Let $\Pi : H^2(\mathbf{C^k}) \to H^2(\mathbf{C^k}) \ominus P_1 H^2(\mathbf{C^k})$ denote orthogonal projection. Using the linear commutant lifting theorem (CLT) (see [15] for the details), we may choose q_1 such that

$$\|W_1 - P_1 q_1\| = \|\Pi W_1\|.$$

Now given this q_1, we choose (using CLT) q_2 such that

$$\|W_2 - P_2(q_1 \otimes q_1) - P_1 q_2\| = \|\Pi(W_2 - P_2(q_1 \otimes q_1))\|.$$

Inductively, given $q_1, \ldots, q_{n-1}$, set

$$A_n := (W_n - \sum_{2 \leq j \leq n} \sum_{i_1 + \ldots + i_j = n} P_j(q_{i_1} \otimes \ldots \otimes q_{i_j}))$$

for $n \geq 2$. Then from the CLT, we may choose q_n such that

$$\|A_n - P_1 q_n\| = \|\Pi A_n\|. \tag{2}$$

We now come to the key point on the convergence of the iterative commutant lifting method.

Proposition 1 *With the above notation, let $q^{(1)} := q_1 + q_2 + \dots$ Then $q^{(1)} \in C_l$.*

Note that given any $q \in C_l$, we can apply the iterative commutant lifting procedure to $W - P \circ q$. Now set

$$S_\Pi(q)(\rho) := \sum_{n=1} \rho^n \|\Pi(W - P \circ q)_n\|.$$

Clearly, $S_\Pi(q) \le S(q)$ (as functions). We can now state the following result whose proof is immediate from the above discussion:

Proposition 2 *Given $q \in C_l$, there exists $\tilde{q} \in C_l$, such that $S(\tilde{q}) \equiv S_\Pi(q)$. Moreover $\tilde{q}$ may be constructed from the iterated commutant lifting procedure.*

Moreover, we easily have the following result:

Proposition 3 *q is partially optimal if and only if $S(q) \cong S_\Pi(q)$.*

We can now summarize the above discussion with the following:

Theorem 1 *For given P and W as above, any $q \in C_l$ is either partially optimal or can be ameliorated by a partially optimal compensating parameter.*

It is important to emphasize that a partially optimal compensating parameter need not be optimal in the sense of Definition 4(i). Basically, what we have shown here is that using the iterated commutant lifting procedure, we can ameliorate any given design. The question of optimality will be considered in the next section.

6 Optimal Compensators

In this section we will derive our main results about optimal compensators. Basically, we will show that in the single input / single output setting, the iterated commutant lifting procedure leads to an optimal design. Once more we refer the interested reader to [9] and [11] for all of the proofs. We begin with the following:

Theorem 2 *There exist optimal compensators.*

For the construction of the optimal compensator in Theorem 3 below, we will need one more technical result. Accordingly, we will need to set-up a bit more notation. First set $H^2 := H^2(\mathbf{C})$, and $H^\infty := H^\infty(\mathbf{C})$ (the space of bounded analytic complex-valued functions on the unit disc). Let $m \in H^\infty$ be a nonconstant inner function, let $\Pi_1 : H^2 \to H^2 \ominus mH^2 =: H(m)$ denote orthogonal projection, and set $T := \Pi_1 S|H(m)$, where S is the canonical unilateral shift on H^2. (T is the compressed shift.) For H a complex separable Hilbert space, let $S_\infty : H \to H$ denote a unilateral shift, i.e. an isometric operator with no unitary part. This means that $S_\infty^{*n} \to 0$ for all $h \in H$ as $n \to \infty$. (See [15].) We can now state the following generalization of a nice result due to Sarason:

Lemma 1 *Notation as above. Let $A : H \to H^2 \ominus mH^2$ be a bounded linear operator which attains its norm, i.e. such that there exists $h_o \in H$ with $\|Ah_o\| = \|A\|\|h_o\| \neq 0$. Suppose moreover that*

$$AS_\infty = TA.$$

Then there exists a unique minimal intertwining dilation B of A, i.e. an operator $B : H \to H^2$ such that $BS_\infty = SB$, $\|A\| = \|B\|$, and $\Pi_1 B = A$.

We now come to the main result of this section:

Theorem 3 *Let W and P be single SISO locally stable operators, with W the weight and P the plant. Suppose that ΠW_j is compact for $j \ge 1$ and ΠP_k is compact for $k \ge 2$. ($\Pi : H^2 \to H^2 \ominus P_1 H^2$ denotes orthogonal projection.) Let q_{opt} be a partially optimal compensating parameter as constructed by the iterated commutant lifting procedure. Then q_{opt} is optimal.*

Corollary 1 *Let P and W be locally stable and SISO, with linear part P_1 rational. Then the partially optimal compensating parameter q_{opt} constructed by the iterated commutant lifting procedure is optimal.*

7 Nonlinear Mixed Sensitivity

The key point of this section is to show that for stable plants, we may reduce a nonlinear version of the mixed sensitivity problem to a linear 2-block problem (via anyone's favorite technique), and then to a nonlinear one-block problem whose solution has been outlined above. We follow the treatment from Hitay Ozbay's 1989 Ph.D. thesis [14] for our discussion of the mixed sensitivity problem.

Consider the feedback configuration in Figure 1. Here P is the plant to be controlled, C denotes the controller, r the reference signal, n the measurement noise, e the tracking error, and y the output of the plant. The sensitivity operator $S = (I + P \circ C)^{-1}$ is the the mapping from the noise input n to the tracking error e while $I - S$ is the mapping from n to the output y. The norm of the sensitivity operator relates to tracking error while the norm of the complementary sensitivity $I - S$ relates to the stability margin. The problem of mixed sensitivity minimization problem is to minimize the norm of the operator

$$\begin{pmatrix} W_1 S \\ W_2(I - S) \end{pmatrix}.$$

This operator combines both S and $I - S$, with the weights chosen so as to trade-off between conflicting design specifications.

With $C = q \circ (I - P \circ Q)^{-1}$, $Q \in C_l$, as before, the above mixed sensitivity operator becomes

$$A := \begin{pmatrix} W_1 \\ 0 \end{pmatrix} - \begin{pmatrix} W_1 \\ W_2 \end{pmatrix} PQ.$$

Compute an outer function $G \in H^\infty$ and $X, Y \in H^\infty$ such that $W_1^* W_1 + W_2^* W_2 = G^* G$, $X^* X + Y^* Y = I$, and $X^* W_1 + Y^* W_2 = 0$.

Then $M := \begin{pmatrix} W_1 G^{-1} & X \\ W_2 G^{-1} & Y \end{pmatrix}$ is a square inner matrix. Let now B be an inner function so that both $W := BG^* W_1{}^* W_1$, and $F := BX^*$ are in H^∞. Also, let $G_i G_o$ be an inner/outer factorization of GP, and finally define $R := BG_i$ and $q := G_o Q$. Then

$$\|A\| = \|BM^* \begin{pmatrix} W_1 \\ 0 \end{pmatrix} - \begin{pmatrix} W_1 \\ W_2 \end{pmatrix} PQ\| = \|\begin{pmatrix} W - Rq \\ F \end{pmatrix}\|$$

$$= \|\begin{pmatrix} W_1 \\ 0 \end{pmatrix} - \begin{pmatrix} W_1 \\ W_2 \end{pmatrix} PQ\| = \|\begin{pmatrix} W - Rq \\ F \end{pmatrix}\|.$$

Note that W and F depend solely on the weights W_1 and W_2 and thus can be assumed to be linear. Further, note that

$$\|A_n\| = \|(BM^* A)_n\| = \|\left(\begin{pmatrix} W - Rq \\ F \end{pmatrix}\right)_n\|.$$

So we may work with the latter and at the end obtain the optimal $Q = G_o{}^{-1} q$. Minimizing the norm of the linear term $\begin{pmatrix} W - R_1 q_1 \\ F \end{pmatrix}$ is an ordinary 2-block problem. As for $n > 1$,

$$A_n = \begin{pmatrix} (-Rq)_n \\ 0 \end{pmatrix}$$

whose norm can be minimized following the commutant lifting method discussed earlier.

8 Example and Conclusion

We conclude this note with a brief example on our design method. We are planning a much longer paper with a number of design examples illustrating our methods. Consider the nonlinear function $f = (f_1, f_2) : \mathbf{R}^2 \to \mathbf{R}^2$ defined by

$$f_1(u_1, u_2) := \begin{cases} u_1 \text{ for } u_1^2 + u_2^2 \leq 1; \\ \sin(\arctan(u_1/|u_2|)) \text{ for } u_1^2 + u_2^2 > 1 \end{cases}$$

and

$$f_2(u_1, u_2) := \begin{cases} u_2 \text{ for } u_1^2 + u_2^2 \leq 1; \\ \cos(\arctan(|u_1|/u_2)) \text{ for } u_1^2 + u_2^2 > 1 \end{cases}$$

f is a kind of "symmetric saturation."

Consider the linear plant

$$P := \begin{bmatrix} 1/s^2 & 0 \\ 0 & 1/s^2 \end{bmatrix}.$$

We applied our optimal sensitivity procedure to the nonlinear system $\widehat{P} := P \circ f$. (We only very briefly sketch the details here.) It turns out that $\widehat{P}$ is strongly stabilizable by a compensator C which consists of lead networks $\frac{s+\epsilon}{\epsilon s+1}$, $\epsilon < 1$, on the diagonal. This allowed us to write down the necessary nonlinear Bezout identity for which to reduce ourselves to type of problem solved in Section 5 and write down a nonlinear suboptimal compensator. (The compensator is only suboptimal since for opimality we would need $\epsilon = 0$ which would violate the causality constraint.) Making use of the fact that our nonlinearity is odd, and that the linear part of the plant is outer we were able to carry out the relevant computations. Some results of simulations with unit step and unit sinusoidal output disturbances are given below for $\epsilon = .25$. In both cases the signals did "hit" the symmetric saturation so the responses are indeed nonlinear. Of course as the magnitude of the sinusoid increased the attenuation became less evident. At a magnitude of 10, there was very little attenuation for the suboptimal compensator taken with $\epsilon = .25$. Decreasing ϵ to .025 however did force attenuation even in this highly saturated situation. Details of the computations of our nonlinear controller will be given in a separate publication.

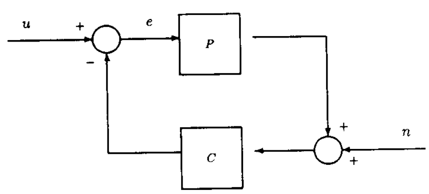
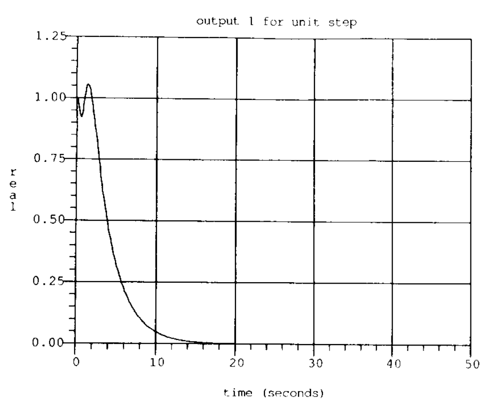


Figure 1: feedback configuration



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